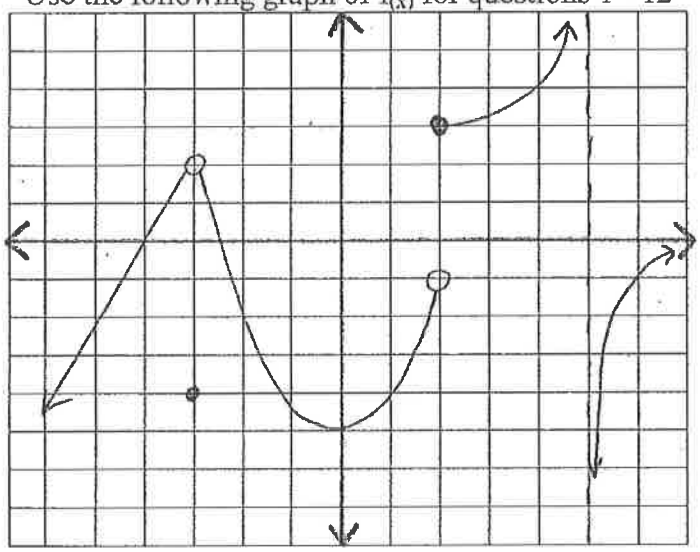
AP Calc AB
Limits, Continuity, Derivative
Test: Review

Use the following graph of $f_{(x)}$ for questions 1 - 12



1. $\lim_{x\to -3^+} f(x) = 2$ 2. $\lim_{x\to -3^-} f(x) = 2$ 3. $\lim_{x\to -3} f(x) = 2$ 4. $f(-3) = -4$

5. $\lim_{x\to 2^+} f(x) = 3$ 6. $\lim_{x\to 2^-} f(x) = -1$ 7. $\lim_{x\to 2} f(x) =$ dne 8. $f(2) = 3$

9. $\lim_{x\to 5^+} f(x) = -\infty$ 10. $\lim_{x\to 5^-} f(x) = \infty$ 11. $\lim_{x\to 5} f(x) =$ dne 12. $f(5) =$ undefined

Find the indicated limit:

13. $\lim_{x\to 2} \frac{x^2-4}{x+5} = 0$

14. $\lim_{x\to -4} |x-7|+2 = 13$

15 $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{2}{3}}{x+3} = \frac{\frac{1}{3}-\frac{2}{3}}{6} = -\frac{1}{18}$

16. $\lim_{x\to -7} \frac{\overset{(\cancel{x+7})(x-4)}{x^2+3x-28}}{\cancel{x+7}} = -11$

17. $\lim_{x\to 36} \frac{\sqrt{x}-6}{x-36} = \frac{(\sqrt{x}+6)}{(\sqrt{x}+6)}$

$\lim_{x\to 36} \frac{\cancel{x-36}}{(\cancel{x-36})(\sqrt{x}+6)}$

$\frac{1}{12}$

18. $\lim_{x\to -5} \frac{\sqrt{x+5}}{2x+10} =$ dne

b/c

$\lim_{x\to -5} \cdot \frac{\sqrt{-5.001+5}}{2(-5.001)+10} \leftarrow$ neg

19. $\lim_{x\to -2^+} \frac{\left(\frac{1}{x}+\frac{1}{2}\right)\cdot 2x}{x+2\ \cdot 2x} =$

$\lim_{x\to -2} \frac{\cancel{2+x}}{(\cancel{x+2})\cdot 2x}$

$-\frac{1}{4}$

20. $\lim_{x\to 7^-} [\![x-3]\!] =$

3

21. $\lim_{x\to 0} \frac{1}{x \csc x} =$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

22. $\lim_{x\to 2^-} \frac{x^2+11x+18}{x-2} = \frac{(+)}{(\text{small} -)}$

$-\infty$

23. $\lim_{x\to -4^+} \frac{x^3-4x}{x^2-16} = \frac{x(x^2-4)}{(x+4)(x-4)}$ $\frac{(-)(+)}{(\text{small} +)(-)}$

∞

24. $\lim_{x\to\infty} \frac{3x-15}{x^2-25} =$ 0

25. $\lim_{x\to-\infty} \frac{x^2-8x+15}{3x+15} =$

$-\infty$

26. $\lim_{x\to\infty} \frac{-3x^2+7x-18}{x^2-4} =$ -3

27. For which x value does $y = \frac{x^3-27}{x^2+5x-24}$ have a vertical asymptote? $\frac{(\cancel{x-3})(x^2+3x+9)}{(x+8)(\cancel{x-3})}$

$x = -8$

28. For which x value does $y = \frac{2x^2-11x-21}{2x^2+13x+15}$ have point discontinuity? $\frac{(\cancel{2x+3})(x-7)}{(\cancel{2x+3})(x+5)}$

$x = -\frac{3}{2}$

29. For which x value does $y = \frac{|x-5|}{x-5}$ have a jump discontinuity?

$x = 5$

30. For which x value does $y = \frac{x+3}{x^2+2x-3}$ have an infinite discontinuity? $\frac{x+3}{(x+3)(x-1)}$

$x = 1$

31. Find the values that make f(x) continuous. Justify your answer.

$$f(x) = \begin{cases} \sqrt{x+3} & x < -2 \\ ax^2 - 7 & x \geq -2 \end{cases}$$

$\sqrt{-2+3} = a(-2)^2 - 7$
$1 = 4a - 7$
$8 = 4a$
$a = 2$

1. $f(-2) = 1$
2. $\lim_{x\to -2} f(x) =$
$\lim_{x\to -2^-} f(x) = 1$ $\lim_{x\to -2^+} f(x)$
3. $f(-2) = \lim_{x\to -2} f(x)$
$\therefore f(x)$ is continuous at $x = -2$

32. Find the values that make f(x) continuous. Justify your answer.

$$f(x) = \begin{cases} 7x+25 & x < -1 \\ ax^2 + b & -1 \leq x < 3 \\ \log_x 9 & x \geq 3 \end{cases}$$

$7(-1)+25 = a(-1)^2 + b$
$18 = a + b$
$a(3)^2 + b = \log_3 9$
$9a + b = 2$

$9a + (18-a) = 2$
$8a + 18 = 2$
$8a = -16$
$a = -2$
$b = 20$

33. Use the limit definition of a derivative to find the following:

$f(x) = 2x^2 - 7x + 1$

A. Find $f'(x) =$ b. Find $f'(3) =$ c. Find $f'(-2) =$

* see additional sheet

34. Describe what the derivative of a function is as it relates to the graph of the function.

- slope of the line tangent - instantaneous rate of change

35. Use the alternate form of the limit definition to find $f'(4) =$ if $f(x) = -x^2 - 5x$.

* see additional sheet

36. Find the equation of the line tangent to the curve $f(x) = -2x^2 - 3x$ at $x = 2$.

* see additional sheet

37. Find the equation of the line orthogonal to the curve $f(x) = \frac{6}{x}$ at $x = 3$.

* see additional sheet

38. Does the function $f(x) = x^4 - 5x^2 + 2$ have a root in the interval [2, 3]?

$f(2) = -2$ by IVT; since $f(2) < 0 < f(3)$, there must

$f(3) = 38$ be a value c such that $f(c) = 0$

39. If $f(x) = x^2 - \sqrt{x+2}$, show there is a number c such that $f(c) = 7$.

$f(2) = 2$ by IVT; since $f(2) < 7 < f(7)$, there must

$f(7) = 46$ be a value c such that $f(c) = 7$

The functions f and g are differentiable for all real numbers, and f is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = g(f(x)) + 4$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	2	1	4	6
4	3	4	9	1
6	8	8	-3	4
8	14	11	2	-3

40. Explain why there must be a value r for $2 < r < 6$ such that $h(r) = 7$

$h(2) = g(f(2)) + 4$ $h(6) = g(f(6)) + 4$ by IVT; since $h(6) < 7 < h(2)$,

$h(2) = 8$ $h(6) = 6$ there must exist some value for r

such that $h(r) = 7$

41. Find the equation of the line normal to $f(x)$ at $x = 4$

point: $(4, 3)$ equation:

$f'(4) = 4$ $y - 3 = -\frac{1}{4}(x-4)$

42. Find the equation of the line tangent to $g(x)$ at $x = 8$

point: $(8, 2)$ equation:

$g'(8) = -3$ $y - 2 = -3(x-8)$

43. Explain why there must be a point where the graph of g has a horizontal tangent line.

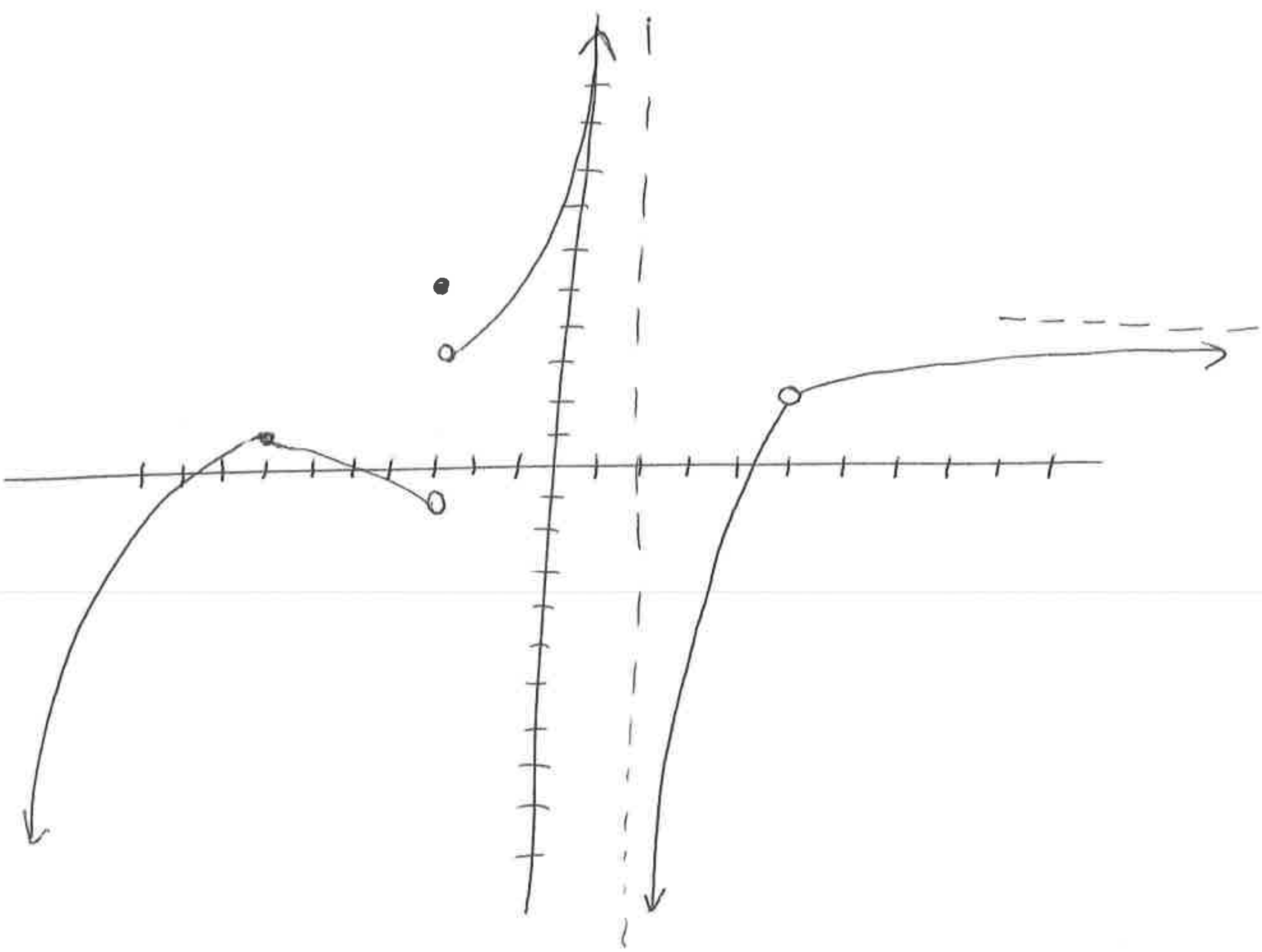
* a horizontal tangent would indicate that $g'(x) = 0$
* since $g'(8) < 0 < g'(6)$, by IVT there must exist some number c such that $g'(c) = 0$

44. Sketch a possible graph for f(x) given the following criteria.

$\lim_{x \to \infty} f(x) = 4$ $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$

$\lim_{x \to 5} f(x) = 2$ $\lim_{x \to -3^-} f(x) = -1$ $\lim_{x \to -3^+} f(x) = 3$ $f(-3) = 5$

$f(-7) = 1$ $f(5)$ *is undefined*



33. $f(x) = 2x^2 - 7x + 1$

a) $\lim_{h \to 0} \frac{2(x+h)^2 - 7(x+h) + 1 - (2x^2 - 7x + 1)}{h}$

$\lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 7x - 7h + 1 - 2x^2 + 7x - 1}{h}$

$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-7x} - 7h \cancel{+1} \cancel{-2x^2} \cancel{+7x} \cancel{-1}}{h}$

$\lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}} - 7\cancel{h}}{\cancel{h}}$

$f'(x) = 4x - 7$

b) $f'(3) = 4(3) - 7 = 5$

c) $f'(-2) = 4(-2) - 7 = -15$

35. $\lim_{x \to 4} \frac{-x^2 - 5x + 36}{x - 4}$

$\lim_{x \to 4} \frac{-(x^2 + 5x - 36)}{x - 4}$

$\lim_{x \to 4} \frac{-(x + 9)\cancel{(x - 4)}}{\cancel{x - 4}}$

$f'(4) = {}^{-}13$

36. $\lim_{x \to 2} \frac{-2x^2 - 3x + 14}{x - 2}$

$\lim_{x \to 2} \frac{-(2x^2 + 3x - 14)}{x - 2}$

$\lim_{x \to 2} \frac{-(2x + 7)(x - 2)}{x - 2}$

$f'(2) = -11$

$f(2) = {}^{-}14$

$y + 14 = -11(x - 2)$

37. $f(3) = 2$

$$\lim_{x \to 3} \frac{\left(\frac{6}{x} - 2\right) \cdot x}{(x-3) \cdot x}$$

$$\lim_{x \to 3} \frac{6 - 2x}{(x-3)x}$$

$$\lim_{x \to 3} \frac{-2\cancel{(x-3)}}{\cancel{(x-3)}x}$$

$$\lim_{x \to 3} \frac{-2}{x}$$

$$f'(3) = -2/3$$

$$(y-2) = 3/2(x-3)$$

32. (justification)

$$f(x) = \begin{cases} 7x+25 & x < -1 \\ -2x^2 + 20 & -1 \leq x < 3 \\ \log_x 9 & x \geq 3 \end{cases}$$

$x = -1$:

1. $f(-1) = 18$

2. $\lim_{x \to -1} f(x) = 18$

$\lim_{x \to -1^-} f(x) = 18$ $\lim_{x \to -1^+} f(x) = 18$

3. $f(-1) = \lim_{x \to -1} f(x)$

$\therefore f(x)$ is continuous @ $x = -1$

$x = 3$:

1. $f(3) = 2$

2. $\lim_{x \to 3} f(x) =$

$\lim_{x \to 3^-} f(x) = 2$ $\lim_{x \to 3^+} f(x) = 2$

3. $f(3) = \lim_{x \to 3} f(x)$

$\therefore f(x)$ is continuous @ $x = 3$